KEMENTERIAN
PENDIDIKAN
MALAYSIA

QS 015/1

Matriculation Programme Examination

Semester I

Session 2017/2018

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1. Given matrix $A = \begin{pmatrix} 2 & 3 \\ -2 & 5 \end{pmatrix}$ such that $A^2 + \alpha A + \beta I = 0$, α and β are constants, where I and O are identity matrix and zero matrix of 2×2 respectively. Determine the value of α and β.

2. Solve the equation $3^{2x+1} - (16)3^x + 5 = 0$.

3. The first and three more successive terms in a geometric progression are given as follows:

   7, …, 189, y, 1701, …

   Obtain the common ratio r. Hence, find the smallest integer n such that the n-th term exceeds 10,000.

4. a) Expand $\left(1 - \frac{x}{3}\right)^{\frac{1}{2}}$ in ascending power of x up to the term in x^3 and state the interval of x for which the expansion is valid.

   b) From part 4(a), express $\sqrt{9 - 3x}$ in the form of $a\left(1 - \frac{x}{3}\right)^{\frac{1}{2}}$, where a is an integer.

   c) Hence, by substituting the suitable value of x, approximate $\sqrt{8.70}$ correct to two decimal places.

5. Solve the equation $3\, log_9\, x = (log_3\, x)^2$.

6. Given a complex number $z = 2 + i$.

   a. Express $\bar{z} - \frac{1}{\bar{z}}$ in the form $a + bi$, where a and b are real numbers.

   b. Obtain $\left|\bar{z} - \frac{1}{\bar{z}}\right|$. Hence, determine the values of real numbers α and β if $\alpha + \beta i = \left|\bar{z} - \frac{1}{\bar{z}}\right| \left(\bar{z} - \frac{1}{\bar{z}}\right)^2$.

7. Find the interval of x for which the following inequalities are true.

   a. $\frac{5}{x+3} - 1 \leq 0$.

   b. $\left|\frac{3x-2}{2x+3}\right| > 2$.

8. Consider functions of $f(x) = (x-2)^2 + 1, x > 2$ and $g(x) = \ln(x+1), x > 0$.

   a. Find $f^{-1}(x)$ and $g^{-1}(x)$, and state the domain and range for each of the inverse function.

   b. Obtain $(g \circ f)(x)$. Hence, evaluate $(g \circ f)(2)$.

9. Given the function $g(x) = \frac{1}{2x-5}$.

   a. Find the domain and range of $g(x)$.

   b. Show that $g(x)$ is a one-to-one function. Hence, find $g^{-1}(x)$.

   c. On the same axis, sketch the graph of $g(x)$ and $g^{-1}(x)$.

   d. Show that $g \circ g^{-1}(x) = x$.

10. Given the system of linear equations as follow:

$2x + 4y + z = 77$

$4x + 3y + 7z = 114$

$2x + y + 3z = 48$

a. Express the system of equations in the form of matrix equation $AX = B$ where $x = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$. Hence, determine matrix A and matrix B.

b. Based on part 10(a), obtain $|A|$.

Hence, find

i. $|P|$ if $PA = I$, where I is an identity matrix $3\ x\ 3$.

ii. $|Q|$ if $Q = (2A)^T$.

iii. $Find\ adjoint\ A$.

Hence, obtain A^{-1} and find the values of $x, y\ and\ z$.

END OF QUESTION PAPER

1. Given matrix $A = \begin{pmatrix} 2 & 3 \\ -2 & 5 \end{pmatrix}$ such that $A^2 + \alpha A + \beta I = 0$, $\alpha \; and \; \beta$ are constants, where I and O are identity matrix and zero matrix of $2 \; x \; 2$ respectively. Determine the value of $\alpha \; and \; \beta$.

SOLUTION

$$A = \begin{pmatrix} 2 & 3 \\ -2 & 5 \end{pmatrix}$$

$$A^2 + \alpha A + \beta I = 0$$

$$\begin{pmatrix} 2 & 3 \\ -2 & 5 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ -2 & 5 \end{pmatrix} + \alpha\begin{pmatrix} 2 & 3 \\ -2 & 5 \end{pmatrix} + \beta\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 4-6 & 6+15 \\ -4-10 & -6+25 \end{pmatrix} + \begin{pmatrix} 2\alpha & 3\alpha \\ -2\alpha & 5\alpha \end{pmatrix} + \begin{pmatrix} \beta & 0 \\ 0 & \beta \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 21 \\ -14 & 19 \end{pmatrix} + \begin{pmatrix} 2\alpha & 3\alpha \\ -2\alpha & 5\alpha \end{pmatrix} + \begin{pmatrix} \beta & 0 \\ 0 & \beta \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -2+2\alpha+\beta & 21+3\alpha \\ -14-2\alpha & 19+5\alpha+\beta \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$21 + 3\alpha = 0$$
$$\alpha = -7$$

$$-2 + 2\alpha + \beta = 0$$
$$-2 + 2(-7) + \beta = 0$$
$$\beta = 16$$

$$\therefore \; \alpha = -7, \; \beta = 16$$

2. Solve the equation $3^{2x+1} - (16)3^x + 5 = 0$.

SOLUTION

$3^{2x+1} - (16)3^x + 5 = 0$

$3^{2x}3^1 - (16)3^x + 5 = 0$

$3.(3^x)^2 - (16)3^x + 5 = 0$

Let $y = 3^x$

$3y^2 - 16y + 5 = 0$

$(3y - 1)(y - 5) = 0$

$(3y - 1) = 0$	$(y - 5) = 0$
$y = \frac{1}{3}$	$y = 5$
$3^x = \frac{1}{3}$	$3^x = 5$
$3^x = 3^{-1}$	$\ln 3^x = \ln 5$
$x = -1$	$x \ln 3 = \ln 5$
	$x = 1.465$

$\therefore x = -1 \; or \; x = 1.465$

3. The first and three more successive terms in a geometric progression are given as follows:

$$7, \ldots, 189, y, 1701, \ldots$$

Obtain the common ratio r. Hence, find the smallest integer n such that the n-th term exceeds 10,000.

SOLUTION

$7, \ldots, 189, y, 1701, \ldots$

$a = 7$

$\frac{y}{189} = \frac{1701}{y}$

$y^2 = 321489$

$y = 567$

$r = \frac{567}{189}$

$r = 3$

$T_n > 10000$

$ar^{n-1} > 10000$

$(7)3^{n-1} > 10000$

$3^{n-1} > 1428.57$

$\ln 3^{n-1} > \ln 1428.57$

$(n-1)\ln 3 > \ln 1428.57$

$(n-1) > 6.61$

$n > 6.61 + 1$

$n > 7.61$

$n = 8$

4. a) Expand $\left(1-\frac{x}{3}\right)^{\frac{1}{2}}$ in ascending power of x up to the term in x^3 and state the interval of x for which the expansion is valid.

b) From part 4(a), express $\sqrt{9-3x}$ in the form of $a\left(1-\frac{x}{3}\right)^{\frac{1}{2}}$, where a is an integer.

c) Hence, by substituting the suitable value of x, approximate $\sqrt{8.70}$ correct to two decimal places.

SOLUTION

a. $$\left(1-\frac{x}{3}\right)^{\frac{1}{2}} = 1+\frac{\left(\frac{1}{2}\right)}{1!}\left(-\frac{x}{3}\right)^1+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(-\frac{x}{3}\right)^2+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(-\frac{x}{3}\right)^3$$

$$= 1-\frac{x}{6}-\frac{1}{8}\left(\frac{x^2}{9}\right)-\frac{3}{48}\left(\frac{x^3}{27}\right)$$

$$= 1-\frac{x}{6}-\frac{x^2}{72}-\frac{3}{48}\left(\frac{x^3}{27}\right)$$

$$= 1-\frac{1}{6}x-\frac{1}{72}x^2-\frac{1}{432}x^3$$

The interval of x for which the expansion is valid:

$$\left|\frac{x}{3}\right| < 1$$

$$-1 < \frac{x}{3} < 1$$

$$-3 < x < 3$$

b. $$\sqrt{9-3x} = (9-3x)^{\frac{1}{2}}$$

$$= 9^{\frac{1}{2}}\left(1-\frac{3x}{9}\right)^{\frac{1}{2}}$$

$$= 3\left(1-\frac{x}{3}\right)^{\frac{1}{2}}$$

c. $$\sqrt{8.70} = \sqrt{9-3(0.01)}$$

$x = 0.01$

$\sqrt{9-3x} = 3\left(1-\frac{x}{3}\right)^{\frac{1}{2}}$

$\sqrt{9-3x} = 3\left[1-\frac{1}{6}x-\frac{1}{72}x^2-\frac{1}{432}x^3\right]$

$\sqrt{9-3(0.01)} = 3\left[1-\frac{1}{6}(0.01)-\frac{1}{72}(0.01)^2-\frac{1}{432}(0.01)^3\right]$

$\sqrt{8.7} = 2.95$

5. Solve the equation $3\,log_9\,x = (log_3\,x)^2$.

SOLUTION

$$3\,log_9\,x = (log_3\,x)^2$$

$$\frac{3log_3\,x}{log_3\,9} = (log_3\,x)^2$$

$$\frac{3log_3\,x}{log_3\,3^2} = (log_3\,x)^2$$

$$\frac{3log_3\,x}{2log_3\,3} = (log_3\,x)^2$$

$$\frac{3log_3\,x}{2} = (log_3\,x)^2$$

$$3log_3\,x = 2(log_3\,x)^2$$

$$Let\ y = log_3\,x$$

$$3y = 2y^2$$

$$2y^2 - 3y = 0$$

$$y(2y-3) = 0$$

$y = 0$ $\qquad$ $2y - 3 = 0$

$y = 0$ $\qquad$ $y = \frac{3}{2}$

$log_3\,x = 0$ $\qquad$ $log_3\,x = \frac{3}{2}$

$x = 3^0$ $\qquad$ $x = 3^{\frac{3}{2}}$

$x = 1$ $\qquad$ $x = 5.196$

6. Given a complex number $z = 2 + i$.

a. Express $\bar{z} - \frac{1}{\bar{z}}$ in the form $a + bi$, where a and b are real numbers.

b. Obtain $\left|\bar{z} - \frac{1}{\bar{z}}\right|$. Hence, determine the values of real numbers α and β if

$$\alpha + \beta i = \left|\bar{z} - \frac{1}{\bar{z}}\right|\left(\bar{z} - \frac{1}{\bar{z}}\right)^2.$$

SOLUTION

(a) $z = 2 + i$

$$\bar{z} - \frac{1}{\bar{z}} = (2 - i) - \frac{1}{2 - i}$$

$$= (2 - i) - \frac{1}{(2 - i)}\frac{(2 + i)}{(2 + i)}$$

$$= (2 - i) - \frac{(2 + i)}{4 + 2i - 2i - i^2}$$

$$= (2 - i) - \frac{(2 + i)}{5}$$

$$= 2 - i - \frac{2}{5} - \frac{1}{5}i$$

$$= \frac{8}{5} - \frac{6}{5}i$$

(b) $\left|\bar{z} - \frac{1}{\bar{z}}\right| = \left|\frac{8}{5} - \frac{6}{5}i\right|$

$$= \sqrt{\left(\frac{8}{5}\right)^2 + \left(\frac{6}{5}\right)^2}$$

$$= \sqrt{\frac{64}{25} + \frac{36}{25}}$$

$$= \sqrt{\frac{100}{25}}$$

$$= \sqrt{4}$$

$$= 2$$

$$\alpha + \beta i = \left|\bar{z} - \frac{1}{\bar{z}}\right|\left(\bar{z} - \frac{1}{\bar{z}}\right)^2$$

$$\alpha + \beta i = 2\left(\frac{8}{5} - \frac{6}{5}i\right)^2$$

$$\alpha + \beta i = 2\left[\left(\frac{8}{5}\right)^2 - 2\left(\frac{8}{5}\right)\left(\frac{6}{5}\right)i + \left(\frac{6}{5}i\right)^2\right]$$

$$\alpha + \beta i = 2\left[\frac{64}{25} - \frac{96}{25}i - \frac{36}{25}\right]$$

$$\alpha + \beta i = 2\left[\frac{28}{25} - \frac{96}{25}i\right]$$

$$\alpha + \beta i = \frac{56}{25} - \frac{192}{25}i$$

$\alpha = \frac{56}{25}$ $\qquad$ $\beta = -\frac{192}{25}$

7. Find the interval of x for which the following inequalities are true.

a. $\frac{5}{x+3} - 1 \leq 0$

b. $\left|\frac{3x-2}{2x+3}\right| > 2$

SOLUTION

a) $\frac{5}{x+3} - 1 \leq 0$

$$\frac{5-(x+3)}{x+3} \leq 0$$

$$\frac{5-x-3}{x+3} \leq 0$$

$$\frac{2-x}{x+3} \leq 0$$

$2 - x = 0$ $\quad$ $x + 3 = 0$

$x = 2$ $\quad$ $x = -3$

	$(-\infty, -3)$	$(-3, 2)$	$(2, \infty)$
$2-x$	+	+	-
$x+3$	-	+	+
$\frac{2-x}{x+3}$	-	+	-

$\{x: x < -3 \ \cup x \geq 2\}$

b) $\left|\frac{3x-2}{2x+3}\right| > 2$

$|x| > a \iff x > a \; or \; x < -a$

$\frac{3x-2}{2x+3} > 2$ $\frac{3x-2}{2x+3} - 2 > 0$ $\frac{(3x-2)-2(2x+3)}{2x+3} > 0$ $\frac{3x-2-4x-6}{2x+3} > 0$ $\frac{-x-8}{2x+3} > 0$	or	$\frac{3x-2}{2x+3} < -2$ $\frac{3x-2}{2x+3} + 2 < 0$ $\frac{(3x-2)+2(2x+3)}{2x+3} < 0$ $\frac{3x-2+4x+6}{2x+3} < 0$ $\frac{7x+4}{2x+3} < 0$

$-x-8=0$ $\qquad$ $2x+3=0$

$x=-8$ $\qquad$ $x=-\frac{3}{2}$

	$(-\infty,-8)$	$\left(-8,-\frac{3}{2}\right)$	$\left(-\frac{3}{2},\infty\right)$
$-x-8$	+	-	-
$2x+3$	-	-	+
$\frac{-x-8}{2x+3}$	-	**+**	-

$$\left(-8,-\frac{3}{2}\right)$$

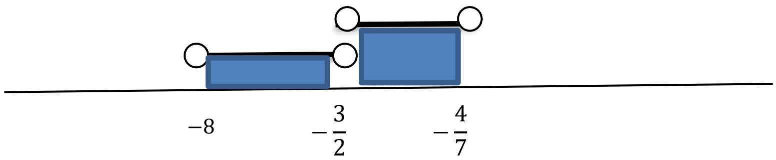
or

$7x+4=0$ $\qquad$ $2x+3=0$

$x=-\frac{4}{7}$ $\qquad$ $x=-\frac{3}{2}$

	$\left(-\infty,-\frac{3}{2}\right)$	$\left(-\frac{3}{2},-\frac{4}{7}\right)$	$\left(-\frac{4}{7},\infty\right)$
$7x+4$	-	-	+
$2x+3$	-	+	+
$\frac{7x+4}{2x+3}$	+	**-**	+

$$\left(-\frac{3}{2},-\frac{4}{7}\right)$$



$$\left(-8,-\frac{3}{2}\right)\cup\left(-\frac{3}{2},-\frac{4}{7}\right)$$

8. Consider functions of $f(x) = (x-2)^2 + 1, x > 2$ and $g(x) = \ln(x+1), x > 0$.

a. Find $f^{-1}(x)$ and $g^{-1}(x)$, and state the domain and range for each of the inverse function.

b. Obtain $(g\ o\ f)(x)$. Hence, evaluate $(g\ o\ f)(2)$.

SOLUTION

$f(x) = (x-2)^2 + 1, \quad x > 2$

$g(x) = \ln(x+1), \quad x > 0.$

(a) Let $y = f^{-1}(x)$

$f(y) = x$

$(y-2)^2 + 1 = x$

$(y-2)^2 = x - 1$

$y - 2 = \sqrt{x-1}$

$y = \sqrt{x-1} + 2$

$f^{-1}(x) = \sqrt{x-1} + 2$

Let $y = g^{-1}(x)$

$g(y) = x$

$\ln(y+1) = x$

$y + 1 = e^x$

$y = e^x - 1$

$g^{-1}(x) = e^x - 1$

$D_{f^{-1}}: (1, \infty)$

$R_{f^{-1}}: (2, \infty)$

$D_{g^{-1}}: (0, \infty)$

$R_{g^{-1}}: (0, \infty)$

(b) $(g\ o\ f)(x)$

$g[f(x)] = g[(x-2)^2 + 1]$

$= \ln\left[[(x-2)^2 + 1] + 1\right]$

$= \ln[(x-2)^2 + 2]$

$(g\ o\ f)(2) = \ln[(2-2)^2 + 2]$

$= \ln[2]$

9. Given the function $g(x) = \frac{1}{2x-5}$.

a. Find the domain and range of $g(x)$.

b. Show that $g(x)$ is a one-to-one function. Hence, find $g^{-1}(x)$.

c. On the same axis, sketch the graph of $g(x)$ and $g^{-1}(x)$.

d. Show that $g \ o \ g^{-1}(x) = x$.

SOLUTION

$$g(x) = \frac{1}{2x-5}$$

(a) $D_g: 2x - 5 \neq 0$

$$x \neq \frac{5}{2}$$

$$D_g: \left(-\infty, \frac{5}{2}\right) \cup \left(\frac{5}{2}, \infty\right)$$

$$R_g: (-\infty, 0) \cup (0, \infty)$$

(b) $g(x) = \frac{1}{2x-5}$

$$g(x_1) = \frac{1}{2x_1 - 5}$$

$$g(x_2) = \frac{1}{2x_2 - 5}$$

Let $g(x_1) = g(x_2)$

$$\frac{1}{2x_1 - 5} = \frac{1}{2x_2 - 5}$$

$$2x_1 - 5 = 2x_2 - 5$$

$$2x_1 = 2x_2$$

$$x_1 = x_2$$

$Since\ x_1 = x_2\ when\ g(x_1) = g(x_2), therefore\ g(x)\ is\ one\ to\ one\ function.$

Let $y = g^{-1}(x)$

$$g(y) = x$$

$$\frac{1}{2y-5} = x$$

$$2y - 5 = \frac{1}{x}$$

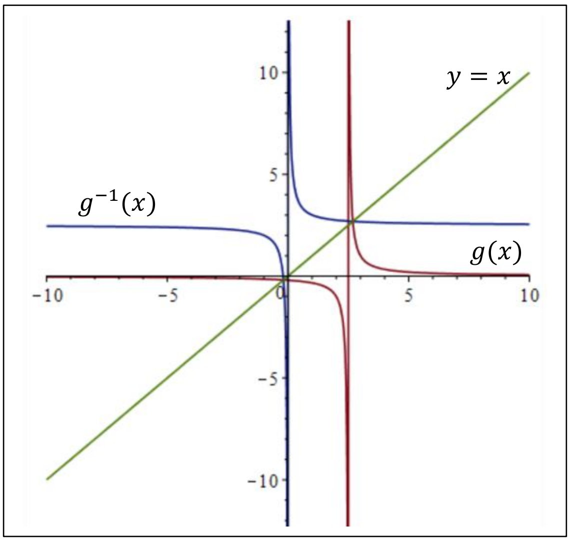
$$2y = \frac{1}{x} + 5$$

$$2y = \frac{1 + 5x}{x}$$

$$y = \frac{1 + 5x}{2x}$$

$$g^{-1}(x) = \frac{1 + 5x}{2x}$$

(c)



(d) $g \; o \; g^{-1}(x) = g[g^{-1}(x)]$

$$= g\left[\frac{1 + 5x}{2x}\right]$$

$$= \frac{1}{2\left[\frac{1 + 5x}{2x}\right] - 5}$$

$$= \frac{1}{\left[\frac{1 + 5x}{x}\right] - 5}$$

$$= \frac{1}{\left[\frac{1 + 5x - 5x}{x}\right]}$$

$$= \frac{1}{\left[\frac{1}{x}\right]}$$

$$= x$$

10. Given the system of linear equations as follow:

$$2x + 4y + z = 77$$
$$4x + 3y + 7z = 114$$
$$2x + y + 3z = 48$$

a. Express the system of equations in the form of matrix equation $AX = B$ where $x = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$. Hence, determine matrix A and matrix B.

b. Based on part 10(a), obtain $|A|$.

Hence, find

i. $|P|$ if $PA = I$, where I is an identity matrix $3\ x\ 3$.

ii. $|Q|$ if $Q = (2A)^T$.

iii. $Find\ adjoint\ A$.

Hence, obtain A^{-1} and find the values of $x, y\ and\ z$.

SOLUTION

$$2x + 4y + z = 77$$
$$4x + 3y + 7z = 114$$
$$2x + y + 3z = 48$$

(a)

$$\begin{pmatrix} 2 & 4 & 1 \\ 4 & 3 & 7 \\ 2 & 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 77 \\ 114 \\ 48 \end{pmatrix}$$

$$AX = B$$

$$A = \begin{pmatrix} 2 & 4 & 1 \\ 4 & 3 & 7 \\ 2 & 1 & 3 \end{pmatrix}; \qquad B = \begin{pmatrix} 77 \\ 114 \\ 48 \end{pmatrix}$$

(b)

$$|A| = +(2)\begin{vmatrix} 3 & 7 \\ 1 & 3 \end{vmatrix} - (4)\begin{vmatrix} 4 & 7 \\ 2 & 3 \end{vmatrix} + (1)\begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix}$$

$$= 2(9 - 7) - 4(12 - 14) + (4 - 6)$$

$$= 2(2) - 4(-2) + (-2)$$

$$= 10$$

(i) $PA = I$

$P = A^{-1}$

$|P| = |A^{-1}|$

$= \frac{1}{|A|}$

$= \frac{1}{10}$

$If\ AB = I\ then\ A = B^{-1}$

$|A^{-1}| = \frac{1}{|A|}$

(ii) $Q = (2A)^T$

$|Q| = |(2A)^T|$

$= 2^3|A^T|$

$= 8|A|$

$= 8(10)$

$= 80$

$(kA)^T = kA^T$

$If\ A\ is\ n\ x\ n\ matrix, then\ |kA| = k^n|A|$

$|A|^T = |A|$

(iii) $adj\ (A) = C^T$

$$A = \begin{pmatrix} 2 & 4 & 1 \\ 4 & 3 & 7 \\ 2 & 1 & 3 \end{pmatrix}$$

$$Cofactor, C = \begin{pmatrix} +\begin{vmatrix} 3 & 7 \\ 1 & 3 \end{vmatrix} & -\begin{vmatrix} 4 & 7 \\ 2 & 3 \end{vmatrix} & +\begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix} \\ -\begin{vmatrix} 4 & 1 \\ 1 & 3 \end{vmatrix} & +\begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} & -\begin{vmatrix} 2 & 4 \\ 2 & 1 \end{vmatrix} \\ +\begin{vmatrix} 4 & 1 \\ 3 & 7 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 4 & 7 \end{vmatrix} & +\begin{vmatrix} 2 & 4 \\ 4 & 3 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 2 & -2 \\ -11 & 4 & 6 \\ 25 & -10 & -10 \end{pmatrix}$$

$adj\ (A) = C^T$

$$= \begin{pmatrix} 2 & 2 & -2 \\ -11 & 4 & 6 \\ 25 & -10 & -10 \end{pmatrix}^T$$

$$= \begin{pmatrix} 2 & -11 & 25 \\ 2 & 4 & -10 \\ -2 & 6 & -10 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|}\ adj(A)$$

$$= \frac{1}{10}\begin{pmatrix} 2 & -11 & 25 \\ 2 & 4 & -10 \\ -2 & 6 & -10 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2}{10} & \frac{-11}{10} & \frac{25}{10} \\ \frac{2}{10} & \frac{4}{10} & \frac{-10}{10} \\ \frac{-2}{10} & \frac{6}{10} & \frac{-10}{10} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{5} & \frac{-11}{10} & \frac{5}{2} \\ \frac{1}{5} & \frac{2}{5} & -1 \\ \frac{-1}{5} & \frac{3}{5} & -1 \end{pmatrix}$$

$AX = B$

$A^{-1}AX = A^{-1}B$

$IX = A^{-1}B$

$X = A^{-1}B$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{1}{5} & \frac{-11}{10} & \frac{5}{2} \\ \frac{1}{5} & \frac{2}{5} & -1 \\ \frac{-1}{5} & \frac{3}{5} & -1 \end{pmatrix} \begin{pmatrix} 77 \\ 114 \\ 48 \end{pmatrix}$$

$$= \begin{pmatrix} 10 \\ 13 \\ 5 \end{pmatrix}$$

$\therefore x = 10, \; y = 13, \; z = 5$